Math 230: Applied Calculus II Quiz #2 Solutions

Week #3

(1) This first problem requires one of the following three **trigonometric identities**

$\tan^2\theta = \sec^2\theta - 1$ | $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ | $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

For each version, this gives

$$\begin{aligned} &\int_0^{\frac{\pi}{4}} 3\tan^2 5x\,dx \\ =& \int_0^{\frac{\pi}{4}} 3\left(\sec^2 5x - 1\right)\,dx \\ =& \int_0^{\frac{\pi}{4}} 3\sec^2 5x - 3\,dx \\ =& \frac{3}{5}\tan 5x - 3x\Big|_0^{\frac{\pi}{4}} \\ =& \left[\frac{3}{5}\tan\left(\frac{5\pi}{4}\right) - \frac{3\pi}{4}\right] - 0 \\ =& \frac{3}{5} - \frac{3\pi}{4} \approx -1.7562 \end{aligned}$$

$$\begin{aligned} &\int_0^{\frac{\pi}{3}} 4\cos^2 2x\,dx \\ =& 4\int_0^{\frac{\pi}{3}} \frac{1}{2}(1+\cos 4x)\,dx \\ =& 2\int_0^{\frac{\pi}{3}} (1+\cos 4x)\,dx \\ =& 2\left(x + \frac{1}{4}\sin 4x\right)\Big|_0^{\frac{\pi}{3}} \\ =& \left[\frac{2\pi}{3} + \frac{1}{2}\sin\left(\frac{4\pi}{3}\right)\right] - 0 \\ =& \frac{2\pi}{3} + \frac{\sqrt{3}}{4} \approx 1.6614 \end{aligned}$$

$$\begin{aligned} &\int_0^{\frac{\pi}{6}} 5\sin^2 3x\,dx \\ =& 5\int_0^{\frac{\pi}{6}} \frac{1}{2}(1-\cos 6x)\,dx \\ =& \frac{5}{2}\int_0^{\frac{\pi}{6}} (1-\cos 6x)\,dx \\ =& \frac{5}{2}\left(x - \frac{1}{6}\sin 6x\right)\Big|_0^{\frac{\pi}{6}} \\ =& \left[\frac{5\pi}{12} - \frac{5}{12}\sin\pi\right] - 0 \\ =& \frac{5\pi}{12} \approx 1.3090 \end{aligned}$$

(2) For each version, u is set equal to $x+k$ for some constant k. This will make $du = dx$ and $x = u - k$. In addition, we'll change the limits of integration appropriately for each version, to give

$$\begin{aligned} & u = x+4 \\ & x = 0 \to u = 4 \\ & x = 5 \to u = 9 \\ & \int_0^5 \frac{x^2}{\sqrt{x+4}}\,dx \\ =& \int_4^9 \frac{(u-4)^2}{\sqrt{u}}\,du \\ =& \int_4^9 \left(u^2 - 8u + 16\right)u^{-\frac{1}{2}}\,du \\ =& \int_4^9 u^{\frac{3}{2}} - 8u^{\frac{1}{2}} + 16u^{-\frac{1}{2}}\,du \\ =& \frac{2}{5}u^{\frac{5}{2}} - 8\left(\frac{2}{3}\right)u^{\frac{3}{2}} + 16(2)u^{\frac{1}{2}}\Big|_4^9 \\ =& \frac{2}{5}\left[9^{\frac{5}{2}} - 4^{\frac{5}{2}}\right] - \frac{16}{3}\left[9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right] \\ +& 32\left[9^{\frac{1}{2}} - 4^{\frac{1}{2}}\right] \\ =& \frac{2}{5}(211) - \frac{16}{3}(19) + 32 \\ =& \frac{226}{15} \approx 15.0667 \end{aligned}$$

$$\begin{aligned} & u = x+9 \\ & x = 0 \to u = 9 \\ & x = 7 \to u = 16 \\ & \int_0^7 \frac{x^2}{\sqrt{x+9}}\,dx \\ =& \int_9^{16} \frac{(u-9)^2}{\sqrt{u}}\,du \\ =& \int_9^{16} \left(u^2 - 18u + 81\right)u^{-\frac{1}{2}}\,du \\ =& \int_9^{16} u^{\frac{3}{2}} - 18u^{\frac{1}{2}} + 81u^{-\frac{1}{2}}\,du \\ =& \frac{2}{5}u^{\frac{5}{2}} - 18\left(\frac{2}{3}\right)u^{\frac{3}{2}} + 81(2)u^{\frac{1}{2}}\Big|_9^{16} \\ =& \frac{2}{5}\left[16^{\frac{5}{2}} - 9^{\frac{5}{2}}\right] - 12\left[16^{\frac{3}{2}} - 9^{\frac{3}{2}}\right] \\ +& 162\left[16^{\frac{1}{2}} - 9^{\frac{1}{2}}\right] \\ =& \frac{2}{5}(781) - 12(37) + 162 \\ =& \frac{152}{5} \approx 30.4 \end{aligned}$$

$$\begin{aligned} & u = x+1 \\ & x = 0 \to u = 1 \\ & x = 3 \to u = 4 \\ & \int_0^3 \frac{x^2}{\sqrt{x+1}}\,dx \\ =& \int_1^4 \frac{(u-1)^2}{\sqrt{u}}\,du \\ =& \int_1^4 \left(u^2 - 2u + 1\right)u^{-\frac{1}{2}}\,du \\ =& \int_1^4 u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + 1u^{-\frac{1}{2}}\,du \\ =& \frac{2}{5}u^{\frac{5}{2}} - 2\left(\frac{2}{3}\right)u^{\frac{3}{2}} + 1(2)u^{\frac{1}{2}}\Big|_1^4 \\ =& \frac{2}{5}\left[4^{\frac{5}{2}} - 1^{\frac{5}{2}}\right] - \frac{4}{3}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] \\ +& 2\left[4^{\frac{1}{2}} - 1^{\frac{1}{2}}\right] \\ =& \frac{2}{5}(31) - \frac{4}{3}(7) + 2 \\ =& \frac{76}{15} \approx 5.0667 \end{aligned}$$