DeVry University Spring 2007

Math 230: Applied Calculus II Quiz #1 Solutions

Week #2

(1) To solve this problem, use the power rule. For each version, this gives

$$\begin{aligned} & \int x^3 - 4 + \frac{3}{x^5}\, dx \\ =& \int x^3 - 4 + 3x^{-5}\, dx \\ =& \frac{1}{4}x^4 - 4x + \frac{3}{-4}x^{-4} + C \\ =& \mathbf{\frac{x^4}{4} - 4x - \frac{3}{4x^4} + C} \end{aligned}$$

$$\begin{aligned} & \int x^4 - 5 + \frac{6}{x^4}\, dx \\ =& \int x^4 - 5 + 3x^{-4}\, dx \\ =& \frac{1}{5}x^5 - 5x + \frac{6}{-3}x^{-3} + C \\ =& \mathbf{\frac{x^5}{5} - 5x - \frac{2}{x^3} + C} \end{aligned}$$

$$\begin{aligned} & \int x^5 - 6 + \frac{7}{x^3}\, dx \\ =& \int x^5 - 6 + 3x^{-3}\, dx \\ =& \frac{1}{6}x^6 - 6x + \frac{7}{-2}x^{-2} + C \\ =& \mathbf{\frac{x^6}{6} - 6x - \frac{7}{2x^2} + C} \end{aligned}$$

(2) This problem, requires a simple substitution. We will change the limits of integration as well.

$$\begin{aligned} u &= -3x^2 \\ du &= -6x\, dx \\ -\frac{1}{6}\, du &= x\, dx \end{aligned}$$

x	**u**
0	0
2	-12

$$\begin{aligned} & \int_0^2 xe^{-3x^2}\, dx \\ =& \int_0^{-12} e^u \left(-\frac{1}{6}\right) du \\ =& -\frac{1}{6}e^u\Big|_0^{-12} \\ =& -\frac{1}{6}\left(e^{-12} - 1\right) \\ =& \mathbf{\frac{1}{6}\left(1 - e^{-12}\right) \approx 0.1666656} \end{aligned}$$

$$\begin{aligned} u &= -2x^2 \\ du &= -4x\, dx \\ -\frac{1}{4}\, du &= x\, dx \end{aligned}$$

x	**u**
0	0
3	-18

$$\begin{aligned} & \int_0^3 xe^{-2x^2}\, dx \\ =& \int_0^{-18} e^u \left(-\frac{1}{4}\right) du \\ =& -\frac{1}{4}e^u\Big|_0^{-18} \\ =& -\frac{1}{4}\left(e^{-18} - 1\right) \\ =& \mathbf{\frac{1}{4}\left(1 - e^{-18}\right) \approx 0.2499999962} \end{aligned}$$

$$\begin{aligned} u &= -4x^2 \\ du &= -8x\, dx \\ -\frac{1}{8}\, du &= x\, dx \end{aligned}$$

x	**u**
0	0
1	-4

$$\begin{aligned} & \int_0^1 xe^{-4x^2}\, dx \\ =& \int_0^{-4} e^u \left(-\frac{1}{8}\right) du \\ =& -\frac{1}{8}e^u\Big|_0^{-4} \\ =& -\frac{1}{8}\left(e^{-4} - 1\right) \\ =& \mathbf{\frac{1}{8}\left(1 - e^{-4}\right) \approx 0.12271} \end{aligned}$$

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Math 230: Applied Calculus II Quiz #2 Solutions

Week #3

(1) To solve this problem, let u be one of the trigonometric functions and substitute for the other trigonometric function using the following derivatives:

$$(\cos x)' = -\sin x,\ (\tan x)' = \sec^2 x, \text{ and } (\sec x)' = \sec x \tan x.$$

In addition, the following trigonometric identities are required:

$$\sin^2 x = 1 - \cos^2 x,\ \sec^2 x = \tan^2 x + 1, \text{ and } \tan^2 x = \sec^2 x - 1$$

For each version, the indefinite integral is evaluated as follows

$$\begin{aligned} u &= \cos x \\ du &= -\sin x\ dx \\ &\int \sin^3 x \cos^6 x\ dx \\ &= \int -\sin^2 x \cos^6 x(-\sin x\ dx) \\ &= \int -(1-u^2)u^6\ du \\ &= \int u^8 - u^6\ du \\ &= \frac{u^9}{9} - \frac{u^7}{7} \\ &= \mathbf{\frac{\cos^9 x}{9} - \frac{\cos^7 x}{7} + C} \end{aligned}$$

$$\begin{aligned} u &= \sec x \\ du &= \sec x \tan x\ dx \\ &\int \tan^3 x \sec^5 x\ dx \\ &= \int \tan^2 x \sec^4 x(\sec x \tan x\ dx) \\ &= \int (u^2-1)u^4\ du \\ &= \int u^6 - u^4\ du \\ &= \frac{u^7}{7} - \frac{u^5}{5} \\ &= \mathbf{\frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C} \end{aligned}$$

$$\begin{aligned} u &= \tan x \\ du &= \sec^2 x\ dx \\ &\int \tan^4 x \sec^6 x\ dx \\ &= \int \tan^4 x \sec^4 x(\sec^2 x\ dx) \\ &= \int u^4(u^2+1)^2\ du \\ &= \int u^8 + 2u^6 + u^4\ du \\ &= \frac{u^9}{9} + \frac{2u^7}{7} + \frac{u^5}{5} \\ &= \mathbf{\frac{\tan^9 x}{9} + \frac{2\tan^7 x}{7} + \frac{\tan^5 x}{5} + C} \end{aligned}$$

(2) This problem requires two steps in order to compute the x-coordinate of the centroid $\overline{x}$. For a function between a and b, first find the area A and then compute the centroid using the formulas

$$A = \int_a^b f(x)\ dx \text{ and } \overline{x} = \frac{1}{A}\int_a^b x f(x)\ dx$$

The areas are shown below

$$\begin{aligned} A &= \int_0^{\frac{\pi}{9}} 2\sin 3x\ dx \\ &= -\frac{2}{3}\cos 3x\Big|_0^{\frac{\pi}{9}} \\ &= -\frac{2}{3}\left(\cos\frac{\pi}{3} + 1\right) \\ &= -\frac{2}{3}\left(\frac{1}{2} - 1\right) \\ &= \mathbf{\frac{1}{3} \approx 0.33333} \end{aligned}$$

$$\begin{aligned} A &= \int_0^{\frac{\pi}{8}} 3\sin 2x\ dx \\ &= -\frac{3}{2}\cos 2x\Big|_0^{\frac{\pi}{8}} \\ &= -\frac{3}{2}\left(\cos\frac{\pi}{4} + 1\right) \\ &= -\frac{3}{2}\left(\frac{\sqrt{2}}{2} - 1\right) \\ &= \mathbf{\frac{6 - 3\sqrt{2}}{4} \approx 0.43934} \end{aligned}$$

$$\begin{aligned} A &= \int_0^{\frac{\pi}{30}} 4\sin 5x\ dx \\ &= -\frac{4}{5}\cos 5x\Big|_0^{\frac{\pi}{30}} \\ &= -\frac{4}{5}\left(\cos\frac{\pi}{6} + 1\right) \\ &= -\frac{4}{5}\left(\frac{\sqrt{3}}{2} - 1\right) \\ &= \mathbf{\frac{4 - 2\sqrt{3}}{5} \approx 0.10718} \end{aligned}$$

To save space, let's use a table to describe the integration by parts. Essentially, let $u = x$ and dv will be the rest.

u	**dv**
$(+)x$	$2\sin 3x$
$(-)1$	$-\frac{2}{3}\cos 3x$
0	$-\frac{2}{9}\sin 3x$

$$\begin{aligned} & \int_0^{\frac{\pi}{9}} x(2\sin 3x)\,dx \\ &= -\frac{2}{3}x\cos 3x + \frac{2}{9}\sin 3x\Big|_0^{\frac{\pi}{9}} \\ &= \frac{2}{9}\left[\left(-\frac{\pi}{3}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right) - 0\right] \\ &= \frac{2}{9}\left(-\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right) \\ &= \frac{1}{27}\left[3\sqrt{3} - \pi\right] \\ \overline{x} &= \frac{1}{A}\int_a^b xf(x)\,dx \\ &= \frac{3}{1}\cdot\frac{1}{27}\left[3\sqrt{3} - \pi\right] \\ &= \mathbf{\frac{1}{9}\left[3\sqrt{3} - \pi\right]} \\ &\approx \mathbf{0.22828} \end{aligned}$$

u	**dv**
$(+)x$	$3\sin 2x$
$(-)1$	$-\frac{3}{2}\cos 2x$
0	$-\frac{3}{4}\sin 2x$

$$\begin{aligned} & \int_0^{\frac{\pi}{8}} x(3\sin 2x)\,dx \\ &= -\frac{3}{2}x\cos 2x + \frac{3}{4}\sin 2x\Big|_0^{\frac{\pi}{8}} \\ &= \frac{3}{4}\left[\left(-\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right) - 0\right] \\ &= \frac{3}{4}\left(-\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}\right) \\ &= \frac{3\sqrt{2}}{32}\left[4 - \pi\right] \\ \overline{x} &= \frac{1}{A}\int_a^b xf(x)\,dx \\ &= \frac{4}{3(2-\sqrt{2})}\cdot\frac{3\sqrt{2}}{32}\left[4 - \pi\right] \\ &= \mathbf{\frac{\sqrt{2}+1}{8}\left[4 - \pi\right]} \\ &\approx \mathbf{0.25905} \end{aligned}$$

u	**dv**
$(+)x$	$4\sin 5x$
$(-)1$	$-\frac{4}{5}\cos 5x$
0	$-\frac{4}{25}\sin 5x$

$$\begin{aligned} & \int_0^{\frac{\pi}{30}} x(4\sin 5x)\,dx \\ &= -\frac{4}{5}x\cos 5x + \frac{4}{25}\sin 5x\Big|_0^{\frac{\pi}{30}} \\ &= \frac{4}{25}\left[\left(-\frac{\pi}{6}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\right) - 0\right] \\ &= \frac{4}{25}\left(-\frac{\pi\sqrt{3}}{12} + \frac{1}{2}\right) \\ &= \frac{4}{75}\left[6 - \pi\sqrt{3}\right] \\ \overline{x} &= \frac{1}{A}\int_a^b xf(x)\,dx \\ &= \frac{5}{4-2\sqrt{3}}\cdot\frac{4}{75}\left[6 - \pi\sqrt{3}\right] \\ &= \mathbf{\frac{2+\sqrt{3}}{30}\left[6 - \pi\sqrt{3}\right]} \\ &\approx \mathbf{0.06949} \end{aligned}$$