Math 230: Applied Calculus II Exam #3A

Weeks #15

Name:_____

Read Each Question Carefully. Make Sure To Answer The Question That Is Asked. YOU MUST SHOW ALL YOUR WORK FOR FULL CREDIT. **A correct answer with no work will be only worth minimal points.** You may use a **calculator** (no TI-89 or higher or any calculator with a QWERTY keyboard) and a **formula sheet** with writing on one side of a standard 8.5"x11" piece of paper. Good Luck! **Note the values of each problem!**

1	20	
2	14	
3	15	
4	15	
5	13	
6	13	
7	20	
Total	110	

(1) You are given the following periodic piecewise function defined below on the interval $0 < x < 2\pi$ and continued periodically.

$$f(t) = \begin{cases} 1 & 0 < t < \frac{2\pi}{3} \\ 2 & \frac{2\pi}{3} < t < \frac{4\pi}{3} \\ 3 & \frac{4\pi}{3} < t < 2\pi \end{cases}.$$

(a) Draw a picture of this function over the interval $[-2\pi, 4\pi]$. (*Hint*: This is 3 periods.)

(b) Calculate the Fourier coefficient a_0.

(c) Determine the Fourier cosine coefficients a_1, a_2 and a_3.

(d) Determine the Fourier sine coefficients b_1, b_2 and b_3.

(e) One of the previous sets are all zero. Explain why this true. Do **not** say "Because the integrals are all zero".

(2) Match the differential equations with the given differential equations. You do **not** need to check **all** solutions against each equation. However, you must **demonstrate** that the solution you chose actually works!

(a) $y'' - 4y' + 4y = 12$; $y(0) = 5$, $y'(0) = 8$	A. $y = x(4\ln x + 8)$
(b) $(2x + y) - xy' = 0$; $y(1) = 3$.	B. $y = x(2\ln x + 3)$
	C. $y = (4x + 2)e^{2x} + 3$
	D. $y = (6x - 1)e^{-2x} + 6$

(3) Solve the **separable** differential equation

$$y^2(e^x + 2) + e^x \frac{dy}{dx} = 0;\ y(0) = 1.$$

(4) Solve the following **second-order homogeneous** differential equation. (*Hint*: Don't bother using Laplace transforms.)

$$2y'' - 7y' + 6y = 0;\ y(0) = 5;\ y'(0) = 9.$$

(5) Find the Laplace transform $F(s)$ of the following functions $f(t)$:

(a) $f(t) = 3 + 5t^2 + 7\sin 3t + 8e^{4t}$

(b) $f(t) = (3 + 5t^2 + 7\sin 3t)e^{4t}$

(6) Find the inverse Laplace transform $f(t)$ of the following functions $F(s)$.

(a) $F(s) = \dfrac{8}{s^5} + \dfrac{9}{s-2} + \dfrac{13s}{s^2+16}$

(b) $F(s) = \dfrac{9s + 10}{s^2 + 5s}$. (*Hint*: Try a partial fractions decomposition.)

(7) Use the method of Laplace transforms to solve the given **nonhomogenous second-order** differential equation.

$$y'' + 25y = 100;\ y(0) = 1;\ y'(0) = 10$$

(*Hint*: Conduct your partial fractions decomposition carefully!)