

Math 221: Basic Statistics Quiz #6A-I

Week #9

Name: _____

SHOW ALL YOUR WORK FOR FULL CREDIT! Each answer should be accurate to **at least four decimal digits**.

- (1) For the standard normal distribution Z , calculate the probability

$$P(-0.25 < Z < 1.33).$$

- (2) If X is normally distributed with mean $\mu = 95$ and standard deviation $\sigma = 5$, calculate the probability

$$P(83 < X < 104).$$

Math 221: Basic Statistics Quiz #6B-I

Week #9

Name: _____

SHOW ALL YOUR WORK FOR FULL CREDIT! Each answer should be accurate to **at least four decimal digits**.

- (1) For the standard normal distribution Z , calculate the probability

$$P(-0.36 < Z < 1.44).$$

- (2) If X is normally distributed with mean $\mu = 105$ and standard deviation $\sigma = 10$, calculate the probability

$$P(87 < X < 109).$$

Math 221: Basic Statistics Quiz #6C-I

Week #9

Name: _____

SHOW ALL YOUR WORK FOR FULL CREDIT! Each answer should be accurate to **at least four decimal digits**.

- (1) For the standard normal distribution Z , calculate the probability

$$P(-0.47 < Z < 1.55).$$

- (2) If X is normally distributed with mean $\mu = 115$ and standard deviation $\sigma = 20$, calculate the probability

$$P(91 < X < 117).$$

Math 221: Basic Statistics Quiz #6A-II

Week #9

Name: _____

SHOW ALL YOUR WORK FOR FULL CREDIT! Each answer should be accurate to **at least four decimal digits**.

(1) During a promotional contest, a soft drink company places winning caps on one of every six bottles. If you purchase one bottle a day, what is the probability that you find your first winning cap on the fourth day? some time after three days? (*Hint*: Geometric distribution!)

(2) It is estimated that an students access any of Professor Chelst's webpages at a rate of 7 per hour. For a given hour, find the probability that **exactly** 10 students will access one of his webpages. (*Hint*: Poisson distribution)

Math 221: Basic Statistics Quiz #6B-II

Week #9

Name: _____

SHOW ALL YOUR WORK FOR FULL CREDIT! Each answer should be accurate to **at least four decimal digits**.

(1) During a promotional contest, a soft drink company places winning caps on one of every five bottles. If you purchase one bottle a day, what is the probability that you find your first winning cap on the fourth day? some time after three days? (*Hint: Geometric distribution!*)

(2) It is estimated that an students access any of Professor Chelst's webpages at a rate of 9 per hour. For a given hour, find the probability that **exactly** 10 students will access one of his webpages. (*Hint: Poisson distribution*)

Math 221: Basic Statistics Quiz #6C-II

Week #9

Name: _____

SHOW ALL YOUR WORK FOR FULL CREDIT! Each answer should be accurate to **at least four decimal digits**.

(1) During a promotional contest, a soft drink company places winning caps on one of every seven bottles. If you purchase one bottle a day, what is the probability that you find your first winning cap on the fourth day? some time after three days? (*Hint*: Geometric distribution!)

(2) It is estimated that an students access any of Professor Chelst's webpages at a rate of 8 per hour. For a given hour, find the probability that **exactly** 10 students will access one of his webpages. (*Hint*: Poisson distribution)

Math 221: Basic Statistics Quiz #6-I Solutions

Week #9

- (1) To solve the problem involving the standard normal distribution
- Z
- , just keep in mind that

$$P(a < Z < b) = P(Z < b) - P(z < a).$$

This gives

$P(Z < -0.25) \approx 0.401294$	$P(Z < -0.36) \approx 0.359424$	$P(Z < -0.47) \approx 0.319178$
$P(Z < 1.33) \approx 0.908241$	$P(Z < 1.44) \approx 0.925066$	$P(Z < 1.55) \approx 0.939429$
$P(-0.25 < Z < 1.33)$	$P(-0.36 < Z < 1.44)$	$P(-0.47 < Z < 1.55)$
$\approx 0.908241 - 0.401294$	$\approx 0.925066 - 0.359424$	$\approx 0.939429 - 0.319178$
$= 0.506947.$	$= 0.565642.$	$= 0.620251.$

- (2) To solve the problem involving the normal distribution
- X
- , we also need the formula to convert
- x
- values to
- z
- values

$$Z = \frac{X - \mu}{\sigma}.$$

This gives

$z_1 = \frac{83 - 95}{5} = -2.4$	$z_1 = \frac{87 - 105}{10} = -1.8$	$z_1 = \frac{91 - 115}{20} = -1.2$
$z_2 = \frac{104 - 95}{5} = 1.8$	$z_2 = \frac{109 - 105}{10} = 0.4$	$z_2 = \frac{117 - 115}{20} = 0.1$
$P(83 < X < 104)$	$P(87 < X < 109)$	$P(91 < X < 117)$
$= P(-2.4 < Z < 1.8)$	$= P(-1.8 < Z < 0.4)$	$= P(-1.2 < Z < 0.1)$
$\approx 0.964070 - 0.008198$	$\approx 0.655422 - 0.035930$	$\approx 0.539828 - 0.115070$
$= 0.955872$	$= 0.619492$	$= 0.424758$

Math 221: Basic Statistics Quiz #6-II Solutions

Week #9

- (1) To solve the first problem, remember the geometric formulas. X is the number of times that one tries up to and including the first success. Also, each success has a probability p and $q = 1 - p$. Then the probability of succeeding on the k -th try and the probability of succeeding after k **or more** failures are

$$P(X = k) = p \cdot q^{k-1} \text{ and } P(X > k) = q^k.$$

For each version, this gives

$p = \frac{1}{6}$ $q = 1 - \frac{1}{6} = \frac{5}{6}$ $P(X = 4) = \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^{(4-1)}$ $= \frac{5^3}{6^4} \approx 0.09645$ $P(X > 3) = \left(\frac{5}{6}\right)^3$ ≈ 0.57870	$p = \frac{1}{5}$ $q = 1 - \frac{1}{5} = \frac{4}{5}$ $P(X = 4) = \left(\frac{1}{5}\right) \left(\frac{4}{5}\right)^{(4-1)}$ $= \frac{4^3}{5^4} \approx 0.1024$ $P(X > 3) = \left(\frac{4}{5}\right)^3$ ≈ 0.512	$p = \frac{1}{7}$ $q = 1 - \frac{1}{7} = \frac{6}{7}$ $P(X = 4) = \left(\frac{1}{7}\right) \left(\frac{6}{7}\right)^{(4-1)}$ $= \frac{6^3}{7^4} \approx 0.08996$ $P(X > 3) = \left(\frac{6}{7}\right)^3$ ≈ 0.62974
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- (2) To solve the second problem, remember that the average number of students accessing a website per hour is λ . If X represents the number of students accessing the websites in a specific hour, then the Poisson formula is

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

For each version, this gives

$\lambda = 7$ $P(X = 10) = \frac{(7)^{10}}{10!} \cdot e^{-7}$ $\approx 0.07098.$	$\lambda = 9$ $P(X = 10) = \frac{(9)^{10}}{10!} \cdot e^{-9}$ $\approx 0.11858.$	$\lambda = 8$ $P(X = 10) = \frac{(8)^{10}}{10!} \cdot e^{-8}$ $\approx 0.09926.$
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