Math 221: Basic Statistics Exam #3 Solutions

Week #15

(1) Convert each $\overline{x}$ value into a Z-value and then find the cumulative probability for each Z-value and then determine the difference between the two values. The main formula to use is

$$Z = \left(\frac{\overline{x} - \mu}{\sigma}\right)\sqrt{n}$$

The calculations are shown below

$$\begin{aligned}
\mu &= 1.535 \\
\sigma &= 0.045 \\
n &= 37 \\
\overline{x}_1 &= 1.530 \\
z_1 &= \left(\frac{1.530 - 1.535}{0.045}\right)\sqrt{37} = -0.6759 \\
P(Z < z_1) &= 24.96\% \\
\overline{x} &= 1.542 \\
z_2 &= \left(\frac{1.542 - 1.535}{0.045}\right)\sqrt{37} = 0.9462 \\
P(Z < z_2) &= 82.80\% \\
P(1.535 < \overline{x} < 1.542) &= 82.80\% - 24.96\% = 57.84\%
\end{aligned}$$

(2) Let's go through the steps to determine a confidence interval. Note that the sample size n is above 30 and we can use the Z-distribution.

$$\begin{aligned}
\overline{x} &= 0.25 \\
S &= 0.06 \\
n &= 58 \\
c &= 90\% \\
Z_c &= 1.645 \\
E &= \frac{(1.645)(0.06)}{\sqrt{58}} = 0.013 \\
\overline{x} - E &= 0.237 \\
\overline{x} + E &= 0.263
\end{aligned}$$

(3) The formula for a minimum sample size is

$$n \geq \left(\frac{Z\sigma}{E}\right)^2$$

$$\begin{aligned}
E &= 0.20 \\
\sigma &= 0.85 \\
c &= 95\% \\
Z_c &= 1.960 \\
n &\geq \left(\frac{(1.960)(0.85)}{0.20}\right)^2 = 69.39 \\
n &= 70
\end{aligned}$$

(4) For this problem remember that you will probably need to sum the values to get $\sum x$ and also sum the squares to get $\sum x^2$, this will allow you to compute the variance and then the standard deviation using the formula

$$S^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

For the confidence interval we'll just supply the final answer to the nearest integer. Since the sample size is small, we must use the T-distribution.

$$\begin{aligned}
n &= 10 \\
\sum x &= 8743 \\
\sum x^2 &= 8,334,383 \\
\overline{x} &= \frac{8743}{10} = 874.3 \\
S^2 &= \frac{8334383 - \frac{(8743)^2}{10}}{10-1} = 76708.68 \\
S &= \sqrt{76708.68} \approx 276.96 \\
c &= 95\% \\
df &= 10 - 1 = 9 \\
T_{95\%,9} &\approx 2.262 \\
E &= \frac{(2.262)(276.96)}{\sqrt{10}} \approx 198.1 \\
\overline{x} - E &\approx 676 \\
\overline{x} + E &\approx 1072
\end{aligned}$$

(5) To construct a confidence interval of a proportion use the formulas

$$\hat{p} = \frac{X}{n} \text{ and } E = Z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

The calculations are shown below and as n is above 30, we use the Z-distribution.

$$\begin{aligned}
n &= 1900 \\
X &= 725 \\
\hat{p} &= \frac{725}{1900} = 0.3816 \\
\hat{q} &= 1 - \hat{p} = 1 - 0. = 0.6184 \\
c &= 99\% \\
Z_c &= 2.576 \\
E &= 2.576\sqrt{\frac{(0.3816)(0.6184)}{1900}} = 0.0287 \\
\hat{p} - E &= 0.3816 - 0.0287 = 0.3529 = 35.29\% \\
\hat{p} + E &= 0.3816 + 0.0287 = 0.4103 = 41.03\%
\end{aligned}$$

(6) This problem only requires that you identify the null hypothesis H_0 and the alternate hypothesis H_a. They are shown below.

$$H_0 : \ p = 0.21$$

$$H_a : \ p \neq 0.21$$

(7) In this problem, we need to determine the correct pair of hypotheses and the relevant information. Then, since we have a large sample, we find the critical Z-value from α. We use the sample data to determine the Z-statistic. Then, we reject the hypothesis if it meets the necessary conditions. Here we reject if $|Z| > Z_\alpha$.

$$\begin{aligned}
H_0 : \ \mu &= 3.4 \\
H_a : \ \mu &\neq 3.4 \textbf{ two-tailed!} \\
n &= 70 \\
\overline{x} &= 3.2 \\
S &= 1.13 \\
\alpha &= 0.10 \\
Z_\alpha &= 1.645 \\
Z &= \left(\frac{3.2 - 3.4}{1.13}\right)\sqrt{70} = -1.481 \\
|-1.481| &< 1.645 \textbf{ Keep } \mathbf{H_0}
\end{aligned}$$

(8) We do the same thing in this problem. However, as the sample is small, we use the T-distribution instead of the Z-distribution. Also, we reject H_0 only if $T < -T_{\alpha,df}$.

$$
\begin{aligned}
H_a : \mu &< 5.0 \textbf{ left-tailed} \\
H_0 : \mu &\geq 5.0 \\
n &= 20 \\
\overline{x} &= 4.1 \\
S &= 1.0 \\
\alpha &= 0.05 \\
df &= 20 - 1 = 19 \\
T_{.05,19} &= 1.729 \\
T &= \left(\frac{4.1 - 5.0}{1.0}\right)\sqrt{20} = -4.025 \\
-4.025 &< -1.729 \textbf{ Reject } \mathbf{H_0}
\end{aligned}
$$

(9) The final problem involves two-sample hypothesis tests and is not covered on our exam.