# Math 191: Probability & Statistics Quiz #2 Solutions

(1) To find the midrange, first locate the maximum and minimum values and use the formula Midrange $= \frac{\text{Min}+\text{Max}}{2}$. For each version, this gives

$$\text{Min} = 325 \quad : \quad \text{Max} = 424 \qquad \text{Midrange} = \frac{325+424}{2} = 374.5.$$

$$\text{Min} = 339 \quad : \quad \text{Max} = 403 \qquad \text{Midrange} = \frac{339+403}{2} = 371.$$

(2) To find the sample mean $\overline{x}$, use the formula $\overline{x} = \frac{\sum x}{n}$. For each version, this gives

$$\sum x = 3732 \qquad \overline{x} = \frac{3732}{10} = 373.2$$

$$\sum x = 3752 \qquad \overline{x} = \frac{3752}{10} = 375.2$$

(3) To find the sample median $\tilde{x}$, first sort the data and locate the 5th and 6th value. For each version, this gives

| | | | | |
|---|---|---|---|---|
| 325 | 334 | 356 | 363 | 364 |
| 374 | 393 | 397 | 402 | 424 |

$$\tilde{x} = \frac{364+374}{2} = 369$$

| | | | | |
|---|---|---|---|---|
| 339 | 363 | 364 | 364 | 366 |
| 369 | 389 | 393 | 402 | 403 |

$$\tilde{x} = \frac{366+369}{2} = 367.5$$

(4) The range R is given by the formula $R = \text{Max} - \text{Min}$. For each version, this gives

$$R = 424 - 325 = 99 \qquad\qquad R = 403 - 339 = 64$$

(5) To find the standard deviation $S$, use the formulas

$$S^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} \text{ and } S = \sqrt{S^2}.$$

For each version, this gives

$$\sum x^2 = 1401496 \qquad S^2 = \frac{1401496 - \frac{(3732)^2}{10}}{9} = 968.178 \qquad S = \sqrt{968.178} = 31.116.$$

$$\sum x^2 = 1411582 \qquad S^2 = \frac{1411582 - \frac{(3752)^2}{10}}{9} = 425.733 \qquad S = \sqrt{425.733} = 20.633.$$

(6) The data is right-skewed if $\tilde{x} > \overline{x}$. For both versions, the data is slightly right-skewed.

$$373.2 > 369 \qquad\qquad 375.2 > 367.5$$

# Math 191: Probability & Statistics Quiz #3 Solutions

(1) To solve this problem, let's note what each part is requesting. The first part is asking about the probability of $V'$. The second is asking about the probability of $V \cup M$ and the last part is asking about the probability of $(V \cup M)' = V' \cap M'$. The formulas for these events are

$$P(V') = 1 - P(V),\ P(V \cup M) = P(V) + P(M) - P(V \cap M), \text{ and } P(V' \cap M') = 1 - P(V \cup M).$$

For each version, this gives

| | |
|---|---|
| $P(V) = 0.55$ | $P(V) = 0.61$ |
| $P(M) = 0.46$ | $P(M) = 0.37$ |
| $P(V \cap M) = 0.12$ | $P(V \cap M) = 0.19$ |
| [**a**] $\mathrm{P}(\mathrm{V}') = 1 - 0.55 = 0.45$ | [**a**] $\mathrm{P}(\mathrm{V}') = 1 - 0.61 = 0.39$ |
| [**b**] $\mathrm{P}(\mathrm{V} \cup \mathrm{M}) = 0.55 + 0.46 - 0.12 = 0.89$ | [**b**] $\mathrm{P}(\mathrm{V} \cup \mathrm{M}) = 0.61 + 0.37 - 0.19 = 0.79$ |
| [**c**] $\mathrm{P}(\mathrm{V}' \cap \mathrm{M}') = 1 - 0.89 = 0.11$ | [**c**] $\mathrm{P}(\mathrm{V}' \cap \mathrm{M}') = 1 - 0.79 = 0.21$ |

(2) To determine a five number summary, the numbers must first be sorted. For each version, this gives

| | | | | |
|---|---|---|---|---|
| \$19.93 | \$24.40 | \$27.94 | \$41.20 | \$41.87 |
| \$46.91 | \$47.28 | \$53.65 | \$61.84 | \$63.72 |

| | | | | |
|---|---|---|---|---|
| \$20.52 | \$26.25 | \$36.26 | \$42.82 | \$43.60 |
| \$49.85 | \$50.10 | \$51.77 | \$53.45 | \$60.11 |

The **five number summary** consists of the **minimum**, **first quartile**, **median**, **third quartile** and **maximum** values. As there are $n = 10$ values, compute the median value by calculating $\frac{10+1}{2} = 5.5$ which indicates that you average the **5th** and **6th** values in the list. Similarly, the quartile values are determined by finding the $\frac{10(25)}{100} = 2.5 \rightarrow$**3rd** and the $\frac{10(75)}{100} = 7.5 \rightarrow$**8th** values in the list. For each version, the five number summary is given below. *The box-plot is omitted.*

$$\begin{aligned} \text{Minimum} &= \$19.93 \\ \mathrm{Q}_1 &= \$27.94 \\ \text{Median} &= \frac{\$41.87 + \$46.91}{2} = \$44.39 \\ \mathrm{Q}_3 &= \$53.65 \\ \text{Maximum} &= \$63.72 \end{aligned}$$

$$\begin{aligned} \text{Minimum} &= \$20.52 \\ \mathrm{Q}_1 &= \$36.26 \\ \text{Median} &= \frac{\$43.60 + \$49.85}{2} = \$46.725 \\ \mathrm{Q}_3 &= \$51.77 \\ \text{Maximum} &= \$60.11 \end{aligned}$$

# Math 191: Probability & Statistics Quiz #4 Solutions

One way to solve this problem is to construct the following table. I've reoriented it to make it easier to view.

| $X$ | $P(X)$ | $F(X)$ | $XP(X)$ | $X^2P(X)$ |
|---|---|---|---|---|
| 1 | 0.2 | 0.2 | 0.2 | 0.2 |
| 2 | 0.4 | 0.6 | 0.8 | 1.6 |
| 3 | 0.3 | 0.9 | 0.9 | 2.7 |
| 4 | 0.1 | 1.0 | 0.4 | 1.6 |
| **Tot.** | 1 | **N/A** | **2.3** | 6.1 |

(1) The cdf F(x) is as listed.

(2) The mean $E(X) = \sum XP(X)$ is just 2.3.

(3) The variance is $V(X) = \sum X^2P(X) - [E(X)]^2$ or $6.1 - (2.3)^2 = .81$

# Math 191: Probability & Statistics Quiz #5 Solutions

(1) To solve this problem, you need the formula for the binomial probability. If $n$ is the number of trials, $x$ is the number of successes and $p$ is the probability of success (and $q = 1 - p$ is the probability of failure), then

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

(a) In the first case, you have $n$ batteries and a probability of $p$ that the voltage is acceptable. The answers for each version of this problem are

$$\begin{aligned} n &= 7 \\ p &= 0.88 \\ q &= 1 - 0.88 = 0.12 \\ P(5) &= \binom{7}{5}(0.88)^5(0.12)^2 \approx 0.15959 \end{aligned}$$

$$\begin{aligned} n &= 6 \\ p &= 0.86 \\ q &= 1 - 0.86 = 0.14 \\ P(5) &= \binom{6}{5}(0.86)^5(0.14)^1 \approx 0.39516 \end{aligned}$$

(b) To solve the next part, realize that a flashlight requires **two** working batteries. This means that the new success and failure probabilities called $p'$ and $q'$ are given by the formulas

$$p' = p^2 \text{ and } q' = 1 - p' = 1 - p^2.$$

There are still a fixed number of flashlights to examine and for each version, this gives

$$\begin{aligned} n &= 7 \\ p' &= (0.88)^2 = 0.7744 \\ q' &= 1 - 0.7744 = 0.2256 \\ P(5) &= \binom{7}{5}(0.7744)^5(0.2256)^2 \approx 0.29766 \end{aligned}$$

$$\begin{aligned} n &= 6 \\ p' &= (0.86)^2 = 0.7396 \\ q' &= 1 - 0.7396 = 0.2604 \\ P(5) &= \binom{6}{5}(0.7396)^5(0.2604)^1 \approx 0.34576 \end{aligned}$$

(2) For this problem, we first identify the mean arrival rate $\mu$ and then use the following Poisson formulas.

$$P(X = 2) = \frac{\mu^2}{2!}e^{-\mu} \text{ and } P(X < 2) = P(X = 0) + P(X = 1) = \left(\frac{\mu^0}{0!} + \frac{\mu^1}{1!}\right) e^{-\mu} = (1 + \mu)e^{-\mu}.$$

(a) For each version, the answer to this part is

$$\begin{aligned} \mu &= 4 \\ P(2) &= \frac{4^2}{2!}e^{-4} = 8e^{-4} \approx 0..14653 \end{aligned}$$

$$\begin{aligned} \mu &= 6 \\ P(2) &= \frac{6^2}{2!}e^{-6} = 18e^{-6} \approx 0..04462 \end{aligned}$$

(b) For the next part, the answers are

$$P(X < 2) = (1 + 4)e^{-4} = 5e^{-4} \approx 0.09158$$

$$P(X < 2) = (1 + 6)e^{-6} = 7e^{-6} \approx 0.01735$$

(3) Finally, when the time is divided in half the new mean arrival rate $\mu'$ is related to the old rate by $\mu' = \frac{\mu}{2}$. This makes the final answers

$$\begin{aligned} \mu' &= \frac{4}{2} = 2 \\ P(2) &= \frac{2^2}{2!}e^{-2} = 2e^{-2} \approx 0.27067 \end{aligned}$$

$$\begin{aligned} \mu' &= \frac{6}{2} = 3 \\ P(2) &= \frac{3^2}{2!}e^{-3} = \frac{9}{2}e^{-3} \approx 0.22404 \end{aligned}$$

# Math 191: Probability & Statistics Quiz #6 Solutions

(1) The solutions to this problem involve using the table and also the following formulas

$$P(Z > a) = 1 - P(Z < a) \text{ and } P(a < Z < b) = P(Z < b) - P(Z < a).$$

The answers for each problem are:

(a) $P(Z < 1.48) = 0.93056$

(b) $P(Z > -2.39) = 1 - 0.00842 = 0.99158$

(c) $P(-2.39 < Z < 1.48) = 0.93056 - 0.00842 = 0.92214$

(d) $P(Z < \mathbf{0.67}) \approx 0.75$. Actually $c = 0.67449$

(2) In this problem, $\mu = 70$ and $\sigma = 3$, the formulas for converting from $X$ to the standard normal distribution $Z$ and back are

$$Z = \frac{X - \mu}{\sigma} = \frac{X - 70}{3} \text{ and } Z = \sigma Z + \mu.$$

(a) You find $P(X < 75)$ as follows

$$\begin{aligned} X = 75 \quad &\to \quad Z = \frac{75 - 70}{3} = 1.6666 \approx 1.67 \\ P(X < 75) \quad &= \quad P(Z < 1.67) = 0.9525 \end{aligned}$$

(b) You find $P(67 < X < 75)$ as follows

$$\begin{aligned} X = 67 \quad &\to \quad Z = \frac{67 - 70}{3} = -1 \\ P(67 < X < 75) \quad &= \quad P(-1 < Z < 1.67) \\ &= \quad 0.9525 - 0.1587 = 0.7938 \end{aligned}$$

(c) To find the upper limit first find the matching $X$-value. The whole calculation proceeds as follows

$$\begin{aligned} P(X < c) = 0.99 \quad &\to \quad c = 2.33 \\ Z \quad &= \quad 3 * (2.33) + 70 = 76.99 \end{aligned}$$

# Math 191: Probability & Statistics Quiz #7 Solutions

(1) To solve these problems, use the formulas for the exponential distribution $X$ with average time $T$ listed below

$$P(X < t) = 1 - e^{-\frac{t}{T}},\ P(X > t) = e^{-\frac{t}{T}} \text{ and } P(t_1 < X < t_2) = e^{-\frac{t_1}{T}} - e^{-\frac{t_2}{T}}.$$

We note that $T = 1.85$. For each part of the problem, this gives

(a) $P(X < 3) = 1 - e^{-\frac{3}{1.85}} \approx 0.802422$

(b) $P(X > 1) = e^{-\frac{1}{1.85}} \approx 0.582433$

(c) $P(1 < X < 3) = e^{-\frac{1}{1.85}} - e^{-\frac{3}{1.85}} \approx 0.384855$

(2) For this problem, identify the values of the mean $\mu$, standard deviation $\sigma$ and the sample size $n$. Then, convert from $\overline{X}$-values to $Z$-values using the formula

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \left(\frac{\overline{X} - \mu}{\sigma}\right)\sqrt{n}.$$

For each part of the problem, this gives

(a)

$$\begin{aligned} X = 49.75 &\quad\rightarrow\quad \frac{49.75 - 50}{1}\sqrt{100} = -2.5 \\ X = 50.25 &\quad\rightarrow\quad \frac{50.25 - 50}{1}\sqrt{100} = 2.5 \\ P(49.75 < \overline{X} < 50.25) &\quad=\quad P(-2.5 < Z < 2.5) \\ &\quad=\quad 0.993790 - 0.006210 = 0.987581 \end{aligned}$$

(b)

$$\begin{aligned} X = 49.75 &\quad\rightarrow\quad \frac{49.75 - 49.8}{1}\sqrt{100} = -0.5 \\ X = 50.25 &\quad\rightarrow\quad \frac{50.25 - 49.8}{1}\sqrt{100} = 4.5 \\ P(49.75 < \overline{X} < 50.25) &\quad=\quad P(-0.5 < Z < 4.5) \\ &\quad=\quad .999997 - 0.308538 = 0.691459 \end{aligned}$$